

# MIXING OF CHARGE $-1/3$ QUARKS AND CHARGED LEPTONS WITH EXOTIC FERMIONS IN $E_6$ <sup>1</sup>

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## ABSTRACT

A way is suggested to understand why the average masses of  $(d, s, b)$  quarks are smaller than those of  $(u, c, t)$  quarks. In contrast to previously proposed mechanisms relying on different Higgs boson vacuum expectation values or different Yukawa couplings, the mass difference is explained as a consequence of mixing of  $(d, s, b)$  with exotic quarks implied by the electroweak-strong unification group  $E_6$ .

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The currently known fermions consist of quarks  $(u, c, t)$  with charge  $2/3$ , quarks  $(d, s, b)$  with charge  $-1/3$ , leptons  $(e, \mu, \tau)$  with charge  $-1$ , and neutrinos  $(\nu_e, \nu_\mu, \nu_\tau)$ . Some proposals address certain broad features of their masses. Specifically:

- (1) The evidence [1, 2] that neutrino masses are non-zero but tiny with respect to those of other fermions may be evidence for large Majorana masses of right-handed neutrinos, which overwhelm Dirac mass terms and lead to extremely small Majorana masses for left-handed neutrinos [3].
- (2) Many unified theories of the electroweak and strong interactions (see, e.g., [4]) imply a relation between the masses of charged leptons and quarks of charge  $-1/3$  at the unification scale. Such a relation does seem to be approximately satisfied for the members  $\tau, b$  of the heaviest family.

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- (3) The larger (average) masses of the  $(u, c, t)$  quarks with respect to the  $(d, s, b)$  quarks could be a consequence of different vacuum expectation values in a two-Higgs-doublet model [5], where two different doublets are responsible for the masses of quarks of different charges. [In such a picture we would view the masses of the lightest quarks, which have the inverted order  $m(d) > m(u)$ , as due, for example, to a radiative effect, and not characteristic of the gross pattern.]

In the present paper we propose another potential source of difference between masses of quarks of different charges, which arises in a unified electroweak theory based on the gauge group  $E_6$  [6, 7]. The fundamental (27-dimensional) representation of this group contains additional quarks of charge  $-1/3$  and additional charged and neutral leptons, but no additional quarks of charge  $2/3$ . We have identified a simple mixing mechanism which can depress the average mass of  $(d, s, b)$  quarks (and charged leptons) with respect to that of  $(u, c, t)$  quarks without the need for different Higgs vacuum expectation values. This mixing can occur in such a way as to have minimal effect on the weak charged-current and neutral-current couplings of quarks and leptons, but offers the possibility of observable deviations from standard couplings if the new states participating in the mixing are not too heavy. This mechanism was first observed in Ref. [8]. Similar mixing with isosinglet quarks was discussed in Ref. [9], but with a different emphasis (including a mechanism for understanding  $m_d > m_u$ ). A related (“seesaw”) effect was used to describe the top quark mass in a particular theory of electroweak symmetry breaking [10].

We first recall some basic features of  $E_6$  and mass matrices, and then describe a scenario in which  $(d, s, b)$  masses (and those of charged leptons) can be depressed by mixing with their exotic  $E_6$  counterparts. Some consequences of the mixing hypothesis are then noted.

The fundamental 27-dimensional representation of  $E_6$  contains representations of dimension 16, 10, and 1 of  $SO(10)$ . We assume there exist three 27-plets, corresponding to the three quark-lepton families. We may regard ordinary matter (including right-handed neutrinos) of a single quark-lepton family as residing in an  $SO(10)$  16-plet, with  $SU(5)$  content  $5^* + 10 + 1$ . The additional (“exotic”) states in the 10-plet and singlet of  $SO(10)$  are summarized in Table I for one family. Here  $I_L$  and  $I_{3L}$  refer to left-handed isospin and its third component.

All the new states are vector-like. They consist of an isosinglet quark  $h^c$  of charge  $1/3$ , a lepton isodoublet  $(E^-, \nu_E)$ , the corresponding antiparticles, and a Majorana neutrino  $n_e$ .

For simplicity we consider only mixings within a single family, which we shall denote  $(u, d, e, \nu_e)$ . We shall discuss only mass matrices of charged fermions. The neutral lepton sector is of potential interest since it contains possibilities for “sterile” neutrinos not excluded by the usual cosmological and accelerator-based experimental considerations [11].

The simplest mass is that of the  $u$  quark, which cannot mix with any other. We can describe its contribution to the Lagrangian (we omit Hermitian conjugates for

Table I: Exotic fermions in a 27-plet of  $E_6$ .

SO(10)	SU(5)	State	$Q$	$I_L$	$I_{3L}$
10	5	$h^c$	1/3	0	0
		$E^-$	-1	1/2	-1/2
		$\nu_E$	0	1/2	1/2
	$5^*$	$h$	-1/3	0	0
		$E^+$	1	1/2	1/2
		$\bar{\nu}_E$	0	1/2	-1/2
1	1	$n_e$	0	0	0

brevity) in terms of a  $2 \times 2$  matrix

$$\mathcal{M}^u = \begin{bmatrix} 0 & m_u \\ m_u & 0 \end{bmatrix} \quad (1)$$

sandwiched between Weyl spinors  $(u^c, u)$  and  $(u^c, u)^T$ . The zeroes reflect charge and baryon number conservation. To diagonalize  $\mathcal{M}^u$  it is most convenient to square it and note that the corresponding eigenvalues  $m_u^2$  come in pairs. The simplest Higgs representation giving rise to  $m_u$  belongs to the  $[27^*, 10, 5^*]$  of  $[E_6, \text{SO}(10), \text{SU}(5)]$ .

The corresponding mass matrix for quarks of charge  $-1/3$  takes account of the possible mixing between non-exotic  $d$  and exotic  $h$  quarks. Its most general form in a basis  $(d^c, d, h^c, h)$  can be written [7]

$$\mathcal{M}^d = \begin{bmatrix} 0 & m_2 & 0 & M_1 \\ m_2 & 0 & m_3 & 0 \\ 0 & m_3 & 0 & M_2 \\ M_1 & 0 & M_2 & 0 \end{bmatrix} . \quad (2)$$

Here small letters refer to  $\Delta I_L = 1/2$  masses, which are expected to be of electroweak scale or less, while large letters refer to  $\Delta I_L = 0$  masses, which can be of any magnitude (including the unification scale). We shall assume  $m_i \ll M_i$ . If the masses in Eq. (2) arise through vacuum expectation values of a Higgs 27\*-plet (the simplest possibility), their transformation properties are summarized in Table II.

Eq. (2) is diagonalized, as before, by squaring it.  $(\mathcal{M}^d)^2$  decomposes into two separate  $2 \times 2$  matrices, referring to the bases  $(d^c, h^c)$  and  $(d, h)$ . For each of these, the eigenvalues  $\lambda_1$  and  $\lambda_2$  satisfy

$$\begin{aligned} \lambda_1 + \lambda_2 &= m_2^2 + m_3^2 + M_1^2 + M_2^2 , \\ \lambda_1 \lambda_2 &= (M_1 m_3 - M_2 m_2)^2 . \end{aligned} \quad (3)$$

Suppose, to begin with, that  $h$  and  $h^c$  pair up to form a Dirac particle with large mass  $M_2 \gg (M_1, m_2, m_3)$ . Then the two eigenvalues are  $\lambda_1 \simeq m_2^2$  and  $\lambda_2 \simeq M_2^2$ ,

Table II: Simplest transformation properties of terms in  $\mathcal{M}^d$ .

Term	SO(10)	SU(5)
$m_2$	10	5
$m_3$	16*	5
$M_1$	16*	1
$M_2$	1	1

corresponding to light and heavy Dirac particles  $d$  and  $h$ , respectively. If we label basis states with zeroes as subscripts, and physical states without subscripts, this solution corresponds to  $d = d_0$ ,  $d^c = d_0^c$ ,  $h = h_0$ ,  $h^c = h_0^c$ .

For the more general case where  $M_1$  is not negligible in comparison with  $M_2$ , we can write

$$\begin{aligned} M_1 &= M \cos \theta \quad , \quad M_2 = M \sin \theta \quad , \\ m_3 &= m \cos \phi \quad , \quad m_2 = m \sin \phi \quad . \end{aligned} \quad (4)$$

Then for  $m \ll M$ , we have

$$\lambda_1 \simeq m^2 \cos^2(\theta + \phi) \quad , \quad \lambda_2 \simeq m^2 \sin^2(\theta + \phi) + M^2 \quad . \quad (5)$$

This is our central result. It is possible to choose  $\theta + \phi$  in such a way that the  $d$  quark mass is arbitrarily small in comparison with  $m^2$ , whose scale is a typical electroweak scale (as in the case of  $m^u$ ). The opposite situation, in which  $u$ -type quarks are lighter than  $d$ -type quarks, is unnatural in the present scheme.

The physical (left-handed)  $(d^c, h^c)$  states are eigenstates of the matrix

$$\mathcal{M}_{d^c, h^c}^2 = \begin{bmatrix} m^2 \sin^2 \phi + M^2 \cos^2 \theta & m^2 \cos \phi \sin \phi + M^2 \cos \theta \sin \theta \\ m^2 \cos \phi \sin \phi + M^2 \cos \theta \sin \theta & m^2 \cos^2 \phi + M^2 \sin^2 \theta \end{bmatrix} \quad . \quad (6)$$

For  $M \gg m$  the approximate eigenstates are

$$d^c \simeq \sin \theta d_0^c - \cos \theta h_0^c \quad , \quad h^c \simeq \cos \theta d_0^c + \sin \theta h_0^c \quad . \quad (7)$$

In the limit  $\theta = \pi/2$  in which  $M_2 \gg M_1$ , leading to a large Dirac mass for the exotic quark  $h$ , one thus has  $d^c = d_0^c$ ,  $h^c = h_0^c$ .

The physical (left-handed)  $(d, h)$  states are eigenstates of the matrix

$$\mathcal{M}_{d, h}^2 = \begin{bmatrix} m^2 & mM \sin(\theta + \phi) \\ mM \sin(\theta + \phi) & M^2 \end{bmatrix} \quad , \quad (8)$$

specifically

$$d \simeq d_0 - (m/M) \sin(\theta + \phi) h_0 \quad , \quad h \simeq (m/M) \sin(\theta + \phi) d_0 + h_0 \quad . \quad (9)$$

Thus, for  $m \ll M$ , there is little mixing between the isosinglet and isodoublet quarks, and hence little potential for violation of unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Some consequences of this mixing have been explored, for example, in Refs. [7, 8, 9, 12]. The mixing parameter  $\zeta \equiv (m/M) \sin(\theta + \phi)$  and the suppression of  $d$ -type masses are both maximal for  $\theta + \phi = \pm\pi/2$ .

Although the present mechanism for lowering the masses of down-type quarks does not require  $h$  quarks to be accessible at present energies, it is interesting to speculate about this possibility. One effect of mixing between an ordinary  $d$ -type quark and its exotic  $h$ -type counterpart is the modification of couplings of the  $b$  quark. The forward-backward asymmetry  $A_{FB}^b$  in  $e^+e^- \rightarrow Z \rightarrow b\bar{b}$ , and the asymmetry parameter  $A_b$  describing the couplings of the  $b$  to the  $Z$ , are slightly different from the values expected in a standard electroweak fit, where

$$g_{bL} = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \quad , \quad g_{bR} = \frac{1}{3} \sin^2 \theta_W \quad , \quad (10)$$

and  $A_b = (g_{bL}^2 - g_{bR}^2)/(g_{bL}^2 + g_{bR}^2)$  is predicted to be 0.935 for  $\sin^2 \theta_W = 0.2316$ . To account for the experimental value [13, 14] of  $A_b = 0.891 \pm 0.017$  while keeping  $g_{bL}^2 + g_{bR}^2$  fixed (since the total rate for  $Z \rightarrow b\bar{b}$  is now in accord with standard model predictions) one must modify both  $g_{bL}$  and  $g_{bR}$  in such a way that  $g_{bL} \delta g_{bL} \simeq -g_{bR} \delta g_{bR}$  [13].

The present scheme does not fill the bill, since it affects only left-handed couplings, mixing an isodoublet  $d$  with an isosinglet  $h$ . We find  $\delta g_{bL} = \zeta^2/2$ ,  $\delta g_{bR} = 0$ . Here we have assumed an unmixed  $Z$ . Severe constraints apply to the mixing of the  $Z$  with a higher-mass  $Z'$  [15].

The production of  $h\bar{h}$  pairs in hadronic collisions should be governed by standard perturbative QCD, which gives a reasonable account of top quark pair production [16]. For the data sample of approximately  $100 \text{ pb}^{-1}$  obtained in  $p\bar{p}$  collisions at a center-of-mass energy of 1.8 TeV in Run I at the Fermilab Tevatron, it should be possible to observe or exclude values of  $m(h)$  well in excess of  $m(t)$  [17]. It may also be possible to produce or exclude  $h$  quarks singly through the neutral flavor-changing interaction at LEP II via the reaction  $e^+e^- \rightarrow Z^* \rightarrow h + (\bar{d}, \bar{s}, \bar{b})$ . Both charged-current decays  $h \rightarrow W + (u, c, t)$  and neutral-current decays  $h \rightarrow Z + (d, s, b)$  should be characterized by multiple leptons and missing energy in an appreciable fraction of events.

The mixing proposed here applies in an almost identical manner to the charged leptons under the replacements  $d^c \rightarrow e^-$ ,  $d \rightarrow e^+$ ,  $h^c \rightarrow E^-$ ,  $h \rightarrow E^+$ . The charged leptons' masses, just like those of the  $d$ -type quarks, thus may be depressed relative to their unmixed values. One could expect small modifications of *right-handed* lepton couplings since one is then mixing an isosinglet  $e$  with an isodoublet  $E$ .

To conclude, we have presented a mechanism which accounts for the depression in the average masses of down-type quarks and charged leptons relative to that of up-type quarks, without the need for differences in Higgs vacuum expectation values or in values of the largest Yukawa coupling for each type of fermion. This mechanism relies on mixings between ordinary fermions and their exotic counterparts in

$E_6$  multiplets. It may be of use in building more realistic models of quark and lepton masses. Although the exotic  $E_6$  fermions need not be accessible to present experimental searches in order for this mechanism to be effective, they could well be observable in forthcoming searches at the Fermilab Tevatron, the LEP II  $e^+e^-$  collider, or the Large Hadron Collider under construction at CERN.

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